

Theorem: - Establishes generalised form of De Morgan's law.

Statement: - If  $\{A_i : i \in I\}$  be an indexed family of subsets of the universal set  $\Omega$ , then

(i) complement of the Union of an indexed family of sets is the intersection of their complements.

$$\text{i.e. } \left( \bigcup_{i \in I} A_i \right)' = \bigcap_{i \in I} A_i'$$

(ii) The complement of the intersection of an indexed family of sets is the Union of their complements.

$$\text{i.e. } \left( \bigcap_{i \in I} A_i \right)' = \bigcup_{i \in I} A_i'$$

Proof: (i) let  $a$  be any element of  $\Omega$

$$\Rightarrow a \in \left( \bigcup_{i \in I} A_i \right)' \Leftrightarrow a \notin \bigcup_{i \in I} A_i$$

$$\Rightarrow a \notin \text{any } A_i \Leftrightarrow a \notin \text{each } A_i'$$

$$\Leftrightarrow a \in \bigcap_{i \in I} A_i'$$

$$\therefore \left( \bigcup_{i \in I} A_i \right)' \subseteq \bigcap_{i \in I} A_i'$$

$$\text{And } \bigcap_{i \in I} A_i' \subseteq \left( \bigcup_{i \in I} A_i \right)'$$

$$\text{Thus } \bigcup_{i \in I} A_i' = \bigcap_{i \in I} A_i$$

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(ii) let  $a$  be any element of  $\Omega$

$$\Rightarrow a \in (\bigcap_{i \in I} A_i)' \Leftrightarrow a \notin \bigcap_{i \in I} A_i$$

$$\Leftrightarrow a \notin A_i \text{ for at least one } i \in I$$

$$\Leftrightarrow a \in A_i' \text{ for at least one } i \in I \Leftrightarrow a \in \bigcup_{i \in I} A_i'$$

$$\therefore (\bigcap_{i \in I} A_i)' \subseteq \bigcup_{i \in I} A_i'$$

$$\text{and } \bigcup_{i \in I} A_i' \subseteq (\bigcap_{i \in I} A_i)'$$

$$\text{Thus } (\bigcap_{i \in I} A_i)' = \bigcup_{i \in I} A_i'$$

Proved